Determining R

Monday, April 17, 2023 9:32 AM

ex 1) $A(x) = \sum_{n \ge 0} n! x^n = 1 + x + 2x^2 + 6x^3 + 24x^4 + 120x^5 + \dots$

what is R? $\longrightarrow$ ratio test: $\lim_{n\to\infty} \left| \frac{\overset{(n+1)n!}{(n+1)!}\, x^{n+1}}{n!\, x^n} \right| = \overset{\infty}{(n+1)} \cdot \overset{0}{x} \longrightarrow \begin{cases} 0 & \text{if } x = 0 \\ \infty & \text{all other } x\text{'s} \end{cases} \longrightarrow \boxed{R = 0}$

ex 2) $A(x) = \sum_{n \ge 0} \frac{x^n}{n!}$ converges for all $x \in \mathbb{R}$

ratio test: $\lim_{n\to\infty} \left| \frac{\frac{x^{n+1}}{(n+1)!}}{\frac{x^n}{n!}} \right| = \frac{x}{(n+1)} \to 0$ for all x $\boxed{R = \infty}$

in fact, if $f(x)$ approximated by $A(x)$ with radius R @ $x=0$ $\longrightarrow$ f', f'', f''' or $\int f\,dx$

<u>thm 21 & 22</u> : differentiation & integration in $|x| < R$ → converges

- if $A(x)$ converges @ $|x| < R$, then so does $A'(x)$
- if $A(x)$ converges @ $|x| < R$, then so does $\int A(x)$

could possibly make easier / R doesn't change

* here A' & $\int A$ done term by term *

geometric series: $x^n = \frac{1}{1-x} \left(\frac{a}{1-r}\right)$

$|x| < 1 \to -1 < x < 1$
$(r < 1)$

* $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ *

ex) $\ln(1-x)$

$\frac{d}{dx} \ln(1-x) = \frac{1}{1-x} \cdot -1$

$= \frac{-1}{1-x}$

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad (-1 < x < 1)$

$\frac{-1}{1-x} = \sum_{n=0}^{\infty} -x^n$

$\int \frac{-1}{1-x}\,dx = \sum_{n=0}^{\infty} \int -x^n\,dx$

$\frac{-\ln|1-x|}{-1} + c = \sum_{n=0}^{\infty} \frac{-x^{n+1}}{n+1} + c$

plug in $x=0$ on both sides / both 0 / don't need $+c$

$\boxed{\ln|1-x| = \sum_{n=0}^{\infty} \frac{-x^{n+1}}{n+1}}$

* solving function into series *